POCKET ANALYTICS FOR ENGINEERS & GEOLOGISTS

Decline Curves

Oil Well Decline

$$q = q_i e^{-rt}$$ Eq.1

This is the *Arps* model, also referred to as the *Exponential decline* model. It defines the well production rate-time relationship, q vs t. qi is the initial well rate, q is the current rate, and r is the fractional decline rate per time t. Just keep all units consistent.

Cumulative Well Production

$$Q = (q_i - q)/r$$ Eq.2

Integrating Eq.1 gives equation 2. Q is the cumulative production at any time corresponding to the last production rate q.

Well Reserves

$$K \sim q_i / r$$ Eq.3

K is the well reserves or EUR.

Field/Play Reserves

$$Q = K/ (1+ae^{-rt})$$ Eq.4

This is the *Logistic decline* model. a is a constant that partially defines the year of peak production, and K is the field reserves or EUR. (K – Q) is the *remaining reserves* and Q / K is the level of *depletion of the reserves*. Eq. 4 has three constants (K, a, and r) and therefore does not have a unique solution. To get around this we take its derivative.

$$dQ/dt) / Q = r (1 - Q / K)$$ or $$q / Q = r (1 - Q / K)$$ Eq. 5

which provides a linear relationship between q / Q, and Q. This constraint allows the establishment of definitive values for K and r, both of which are obtained by simple extrapolation of the stabilized trend line.

Field Peak Oil Rate

$$q_{max} = 287 K^{0.62}$$ Eq.6

Eq. 5 also defines the production rate-time relationship from which a useful empirical algorithm is obtained (Eq.6).

This is the classic power law relationship between reserves and production capacity. qmax is expressed in thousands of b/d and K in Bbo. This algorithm determines the *production capacity of the reserves of upstream projects (PUDs)*. It also provides an estimate of reserves of *mature* fields from their peak production rates.

For *conventional gas*, the equivalent power law relationship is:

$$q_{max} = 0.21\ K^{0.85} \qquad \text{Eq.7}$$

q is expressed in bcfd and K in tcf.

For *unconventional gas*, the relationship is:

$$q_{max} = 0.56\ K^{0.76} \qquad \text{Eq.8}$$

More Useful Algorithms

Half-life of a Well's Production Capacity

$$t_{1/2} = 70 / r \qquad \text{Eq.9}$$

This relationship is obtained directly from Eq. 1. t ½ is the time it takes for the production capacity of a well (q) to drop/decline to half of its initial capacity (qi); r is the decline rate, %. *Examples:* well production rate will drop to 50% of its initial rate in 7 years if its decline rate is 10%/year. Well production rate would drop to 50% its initial rate in one year if its decline rate is 70% per year; these high decline rates are typical of *unconventional* plays. Similarly, at t ½ , the well has produced *half* of its reserves, K.

Field Decline of Unconventional Plays

Unconventionals are characterized by almost continuous drilling and may involve thousands of producing wells which makes it difficult to determine when **reservoir** production has peaked and subsequently when decline begins. An easy solution: Divide field production rates by the corresponding number of active wells to establish production per well., b/d/well. Peak becomes obvious and decline, reserves, etc. can be processed with the Exponential model. This methodology of converting field production from b/d to b/d/well is also very applicable to analyze conventional reservoirs.

Compound Interest

This is one of the fundamentals of finances. It is simply the reverse of decline analysis. The compound interest equation is:

$$q = q_i e^{+rt} \qquad \text{Eq.10}$$

qi is the initial investment or loan, r is the interest rate, and q the amount after t years.
Likewise, the time to *double* an investment or money owed:

$$t_{double} = 70 / r \qquad \text{Eq.11}$$

In-situ Value of Reserves

$$DCF = \sum Cash\ Flows / e^{it}$$ Eq.12

Cash FLow (CF) is gross revenue from the production stream minus operating costs; i is the discount rate.

$$CF = (Eq.1)^* (P-C)$$ Eq.13

P is the well-head price of oil and C production costs ($/b). The SEC requires that 'booked' reserves be calculated with P and C as constants. Inserting Eq. 13 in Eq. 12 and integrating over the life of the project gives the NPV (net present value) of the reserves.

$$NPV = q_i * (P-C) / (r+i)$$ Eq.14

The Value of Reserves

$$V\$ = NPV / K = r * (P-C) / (r+i)$$ Eq.15

Examples: for C = 1/3 P and r = i, the value of reserves V$ = P/3. Such is the case of normal conditions and is commonly referred to as the one-third rule. Historically, oil and gas reserves are sold on average for about 22 – 35% of their respective well-head prices. For older (marginal) fields, costs are higher, C = 1/2 P, and V$ drops to P /4. For in-fill drilling, C= 1/3 P and r = 2i, V$ = 4/9 P which is an improvement over the normal field development case, V$ = P/3. However, this strategy would be justified only if the gain outweighs the incremental drilling costs. This case is typical for tight oil plays.

Reservoir Quality Index (RQI)

$$RQI = k * h * \varnothing * H * Pg$$ Eq.16

RQI is an index that measures the quality of an oil or gas reservoir. It is the product of five reservoir characteristics:

k >>>>Permeability, Darcy (max=1)

h >>>>Net thickness, ft.

∅ >>>Porosity, fraction

H >>>>Heterogeneity--- N/G (net to gross)

Pg >>>Pore-pressure gradient, psi/ft.*

Example. A reservoir with the following characteristics: net thickness of 500 ft., permeability 0.500 D, porosity 0.15, heterogeneity (N/G) 0.60 and pore-pressure gradient of 0.450 psi/ft. has an RQI = 10.

*Note. Heterogeneity for IOR/EOR projects, use Dykstra-Parsons Variance (DPV) where DPV ~ 1 – 3 Ø.

Production Potential of Newly Discovered Reserves

$$q_{max} = 287 * K^{0.62}$$ Eq.17

qmax> Field Production Capacity, 1000 b/d
K>>>>>Prospect's Reserves, Bbo

Peak Oil Well-Productivity

Sandstone Reservoirs $$q_{max} = 4.2 * \ln(RQI) + 6.0$$ Eq.18

Carbonates $$q_{max} = 2.6 * \ln(RQI) + 4.1$$ Eq.19

qmax>>Well Production Capacity, 1000 b/d

Number of Development Wells to be Drilled in a Newly Discovered Reservoir

(Eq. 17) / (Eq. 18 or Eq. 19) Eq.20

Example. A newly discovered field with an estimated 0.100 Bbo of oil reserves has a production potential, qmax ∼ 70,000 b/d in accordance with Eq. 17. The expected well productivity is 15,700 b/d in accordance with Eq. 18. Consequently, the number of development wells to be drilled is 70,000 / 15,700 ∼ 5.

If the reservoir was carbonate, the expected well productivity would be 10,000 b/d (Eq. 19) and the number of development wells to be drilled to be 7.

World's Best (Gold Standard) Carbonate Oil Fields

Reservoir Attributes

Field	Age	Depth ft	Pressure Gradient Pg, psi/ft	°API	Net Pay h ft	PorosityØ %	Permeability k mD	Heterogeneity Net/Gross N/G	Reservoir Quality Index RQI	Peak Well-Productivity b/d/well	OIP Bbo
Saudi Arabia Ghawar, 1948	Jurassic	6,400	0.45	34	200	19	620	0.85	10	**16,500**	**450**
USA Permian Basin, 1920	Permian	8,500	0.38	33	150	12	18	0.75	<1	**500**	**160**
Mexico Cantarell, 1977	Cretaceous	8,500	0.45	22	500	10	>1,000	0.76	17	**20,000**	**70**
Brasil Lula, 2006	Cretaceous	23,000	0.53	29	1,040	20	300	0.60	20	**15,000**	**25**
North Sea Ekofisk, 1971	Cretaceous	10,400	0.68	34	575	35	5	0.80	1	**11,700**	**8**
Kazakstan Karachaganak, 1979	Permian	16,400	0.40	47	1,900	5	15	0.40	<1	**2,600**	**8**
WORLD											7,000

Carbonates account for >60% of all oil discovered globally and they host the world's largest oil (Ghawar) and gas (North Field/South Pars) fields. The North Field/South Pars gas field is *twice* the size of Ghawar! Carbonates are primarily located in the Middle East which accounts for > 70% of the world’s oil and 90% of its gas reserves. Other important locations are North America and Eurasia. In general, for both carbonates and sandstones, an RQI ≥ 1 indicates top reservoirs with OIPs greater than 1 billion barrels of oil and high well-productivities. Tight reservoirs, those with k*Ø as low as 0.001, such as the Permian Basin as well as the Karachaganak are among the exceptions.

De-Risking Exploration – Identifying Gas and Oil Directly from Seismic

If the p-wave velocity (Vp) is < 11,000 ft/s, the option is gas.

For oil, if the water saturation (Swi) is < 40%, the option is oil. How to calculate Swi from seismic, read Publications: Seismic Velocity Triggers An Early Estimate of Oil in Sandstone Prospects.